

## **Dimensioning of Tube Mills**

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**1. INTRODUCTION**

The tube mill is a simple, approved machine, not difficult to operate well and still competitive when compared with more modern type of mills.

This paper deals with the design criteria used to dimension a tube mill. These criteria are guidelines. But there is no complete analytical theory for optimal mill design and, therefore, this topic still remains a matter of experience.

**2. MAIN DESIGN CRITERIA**

**2.1 Length to diameter ratio**

The relation between the mill shell length and the shell internal diameter is the length to diameter ratio  $\lambda$  (fig. 1).

The length to diameter ratio depends on various factors. The most important ones are:

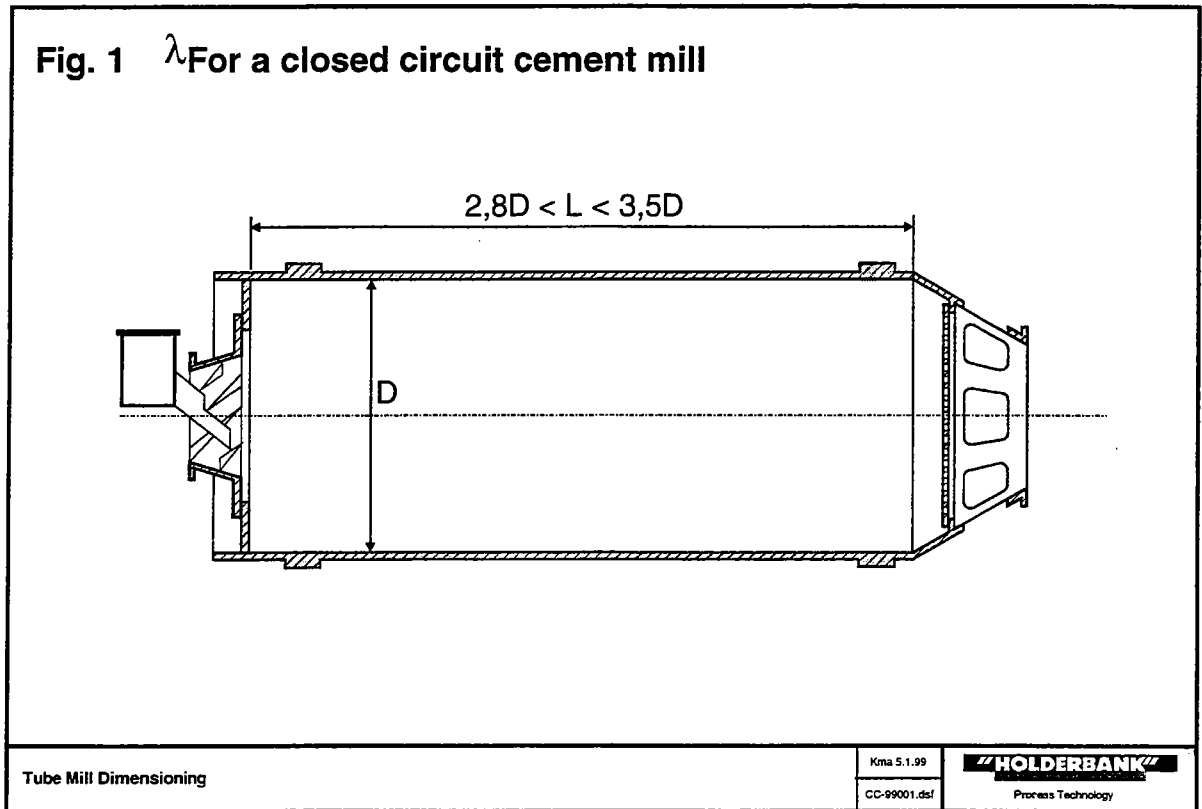
- ◆ Hourly throughput,
- ◆ Type of material to be ground,
- ◆ Fineness of the finished product,
- ◆ Mill in open or closed circuit,
- ◆ Fresh feed size (one stage or two stage grinding),

On one side the hourly throughput depends on the mill diameter. On the other side the fineness at the mill outlet depends mainly on the time the material remains inside the mill, also called retention time. The main factor influencing the retention time is the mill length. Therefore, the ratio of length to diameter of a mill is an important factor for an optimum design of the mill. Table 1 shows guidelines for the length to diameter ratio  $\lambda$ :

**Table 1: Length to diameter ratio  $\lambda$  for different mill systems**

Type of mill system	Ratio $\lambda$	Remarks
<b>Cement mills</b>		
Open circuit mills	3.0 – 6.0	
Closed circuit mills	3.0 – 3.5	large mills: $\lambda = 2.8 - 3.2$ low $\lambda$ yields higher circulating load
Closed circuit mills with pre grinding unit	2.8 – 3.5	
<b>Slurry mills</b>		
Ratio $\lambda$ similar as for cement mills. Max. length 12 - 14 [m]		
<b>Raw mills</b>		
Center discharge mill	2.1 – 2.7	The lower $\lambda$ are applied for large raw mills
Two comp. Mill	2.0 – 2.5	
Single comp. mill	1.7 – 2.2	
Air swept mill	1.5 – 2.0	

Fig. 1  $\lambda$  For a closed circuit cement mill



**2.2 Mill internal dimensions**

The mill shell would only last a few thousand hours if not conveniently protected by shell liners. These internal elements determine the shell internal dimensions.

**2.2.1 Mill useful length**

The mill useful length is defined as the shell length  $L$  reduced by the cumulated width of the head liners and the diaphragms (intermediate and outlet).

Figure 2 shows the mill useful length ( $L_u$ ) for a two-compartment mill.

$L_D$  is the total width of the intermediate diaphragm. For normal applications this value is in the order of 400 [mm].

**2.2.2 Length of compartment**

For mills with two or more compartments there are some guidelines for the length of each different compartment as shown in table 2

Figure 2 shows for a two chamber mill the length of the first ( $L_{u1}$ ) and second ( $L_{u2}$ ) compartments of a two chamber mill.

**Table 2: Length of Grinding Compartments for Different Types of Mills (see Fig. 7)**

Type of mill	[%] of total useful length		
	Comp. I	Comp. II	Comp. III
Two comp. mill	30 - 35	70 - 65	---
Three comp. mill	20	30	50
Center discharge mill	50	50	---

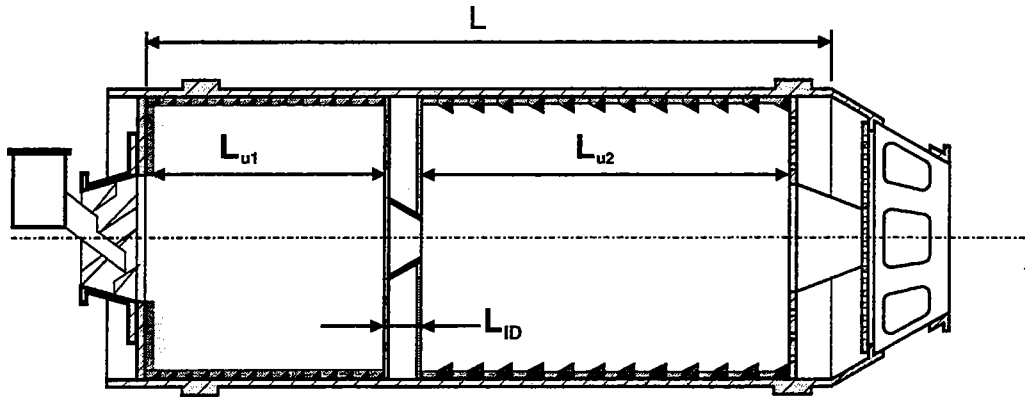
**2.2.3 Mill internal diameter**

The mill internal diameter is defined as the mill shell internal diameter ( $D$ ) reduced by twice the liner average thickness ( $\bar{e}$ ).

As already defined in chapter “tube mills” there are many different types of liners. Figure 3 shows typical liner thicknesses for first (C1) and second (C2) chamber as a function of the mill shell diameter.

Figure 2 & 3

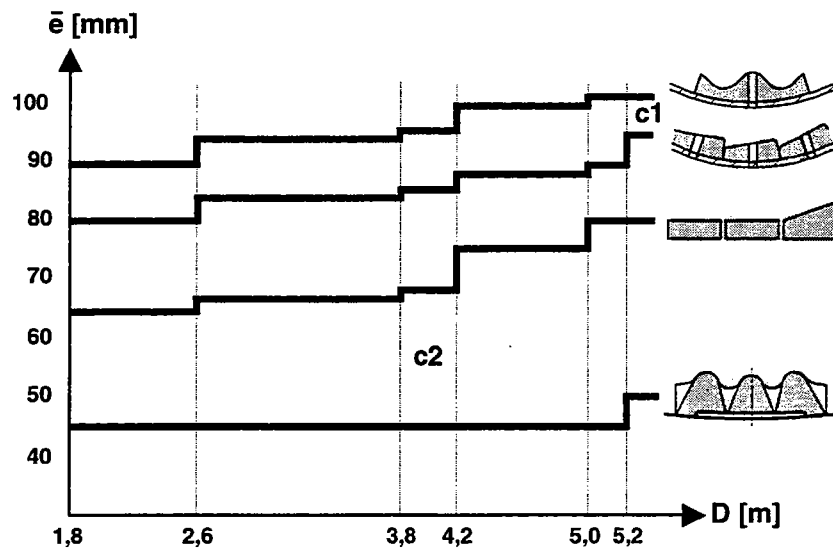
Fig. 2 Useful length for two chambers mill



$$L_u = L_{u1} + L_{u2}$$

$$L_{ID} \sim 400 \text{ [mm]}$$

Fig. 3 Liner thickness



$$D_i = D - 2 \cdot \frac{\bar{e}}{1000} \text{ [mm]}$$

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**2.3 Filling degree**

The filling degree  $f$  is defined as the volume  $V_Q$  of the grinding media charge expressed as a percentage of the total useful mill volume  $V_M$ .

$$f = \frac{V_Q}{V_M} \times 100[\%] \quad (1)$$

Table 3 shows guidelines for filling degree values:

**Table 3: Filling degrees for different types of tube mills**

Type of mill	Filling degree [%]		
	Comp. I	Comp. II	Comp. III
Single comp. mill	27-33	---	---
Two comp. mill	27-33	25-32	---
Three comp. mill	26-32	26-30	23-27
Air swept mill	≈ 26	---	---

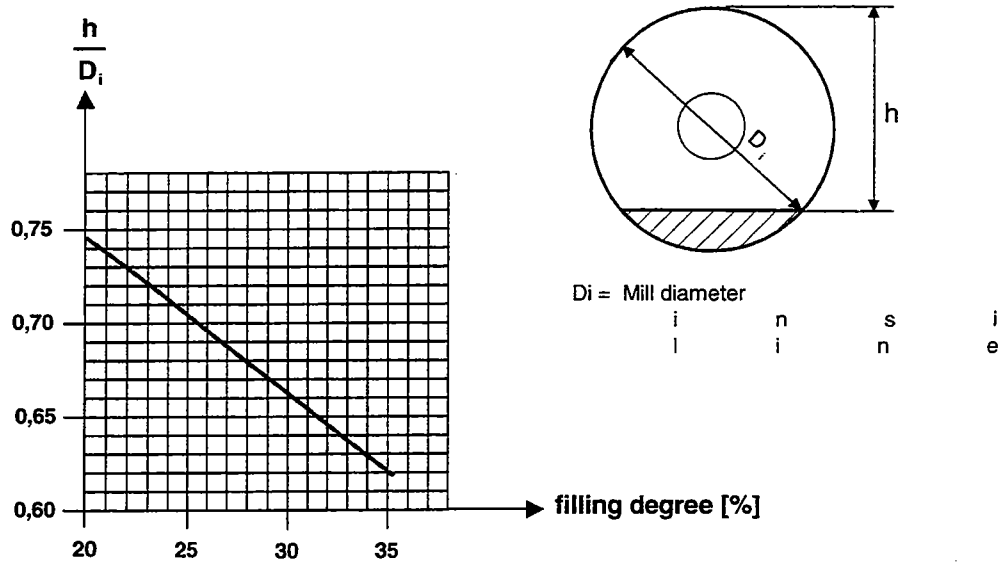
Studies have proven that there is a maximum grinding efficiency for a filling degree of 26 – 28 [%]. Above this value the higher the filling degree the lower the grinding efficiency. Nevertheless, in countries with low electrical energy cost and high market demand, filling degrees of 40 – 45 [%] are used to maximise mill production.

The filling degree in a tube mill can be practically determined by measuring the free heights  $h$  above the grinding charge according to figure 4.

Due to liner design it is not possible to measure the free height accurately. Figure 5 shows an alternative method to determine the free height. Having measured  $d$  and  $h'$ ,  $h$  can be calculated and graph from figure 4 can be used with.

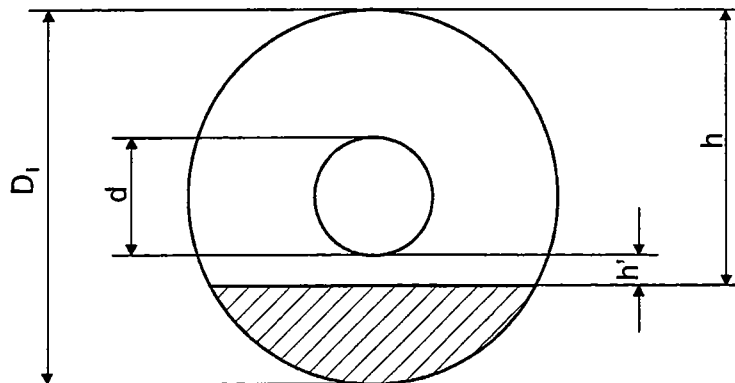
Fig. 4 & 5

**Fig.4 : Filling degree f in function of free height h above grinding media charge**



**Fig.5 : Alternative way to determine the free height**

$$h = \frac{D_i + d}{2} + h'$$



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**2.4 Weight of grinding media**

The weight of the grinding media charge can be calculated as follows:

$$Q = \frac{\pi}{4} \cdot D_i^2 \cdot L_u \cdot \frac{f}{100} \cdot \gamma_Q \quad [t] \quad (2)$$

- where:
- L<sub>u</sub>** = internal length of the mill or compartment [m]
  - f** = filling degree [%]
  - γ<sub>Q</sub>** = grinding media bulk weight [t/m<sup>3</sup>]
  - D<sub>i</sub>** = internal mill diameter (inside lining) [m]
  - Q** = grinding media charge [t/m<sup>3</sup>]

The bulk densities of the grinding media are given in the following table:

**Table 4: Bulk weight of grinding media charge (Figure 6)**

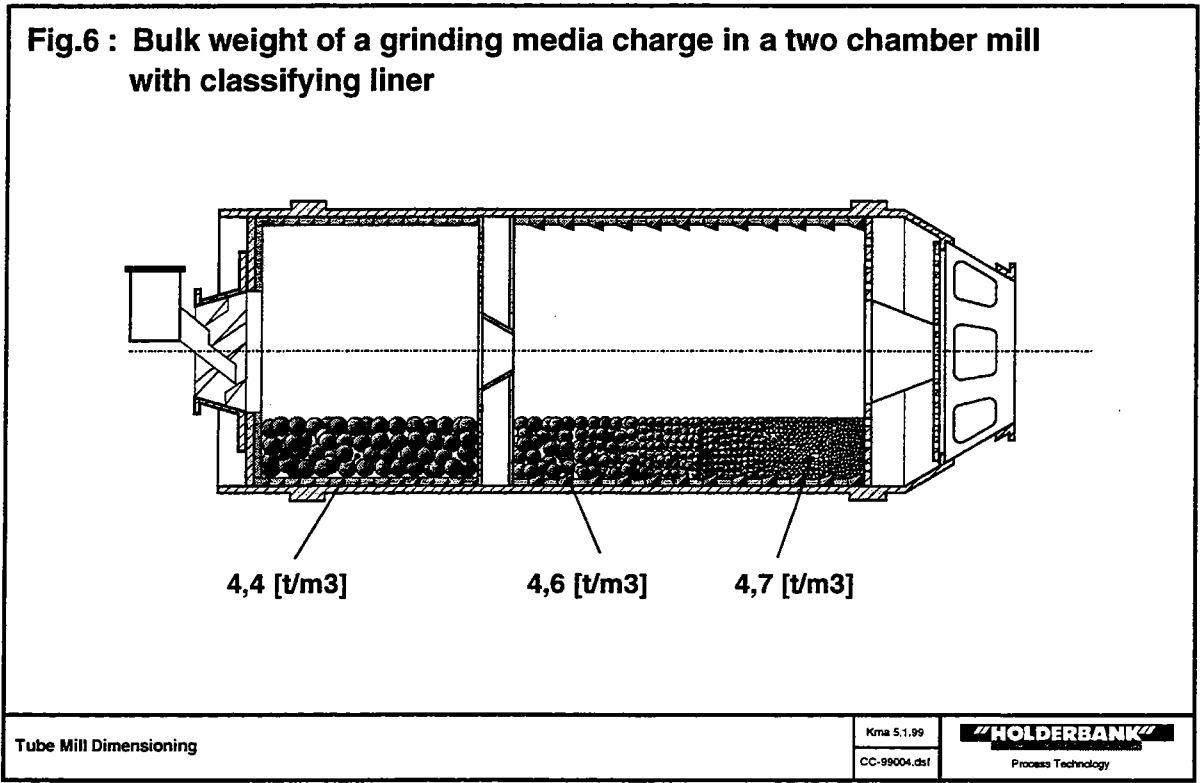
	Ball size Ø [mm]	Bulk weight [t/m <sup>3</sup> ]
Steel balls	100 – 60	4.4
	50 – 30	4.6
	30 – 20	4.7
Cylpebs	30 – 20	4.8

Figure 6 shows the grinding media bulk weight distribution in a two chamber mill equipped with classifying liners in the second chamber.

For calculation 4.4 [t/m<sup>3</sup>] and 4.65 [t/m<sup>3</sup>] are used as grinding media bulk weight for first and second chamber respectively.



**Fig. 6:** Bulk weight of a grinding media charge in a two-chamber mill with classifying liner



## 2.5 Mill speed

The operating speed of the mill can be expressed as a percentage of the critical mill speed. Critical speed is attained when centrifugal force  $F_c$  compels the outer layer of grinding media to rotate with the mill lining.

The critical speed is obtained when the centrifugal force  $F_c$  is equal to the force of gravity  $F_g$ ; i.e. (Fig. 7):

$$F_g - F_c = m \cdot g - m \cdot \frac{D_i}{2} \omega_{crit}^2 = 0 \quad (3)$$

The critical angular speed  $\omega_{crit}$  will then be:

$$\omega_{crit} = \sqrt{\frac{2 \cdot g}{D_i}} \quad (4)$$

calculated as a function of the mill diameter  $D_i$

$$n_{crit} = \frac{30}{\pi} \cdot \sqrt{\frac{2 \cdot g}{D_i}} \quad (5)$$

$$n_{crit} = \frac{42.3}{\sqrt{D_i}} \quad [\text{min}^{-1}] \quad (6)$$

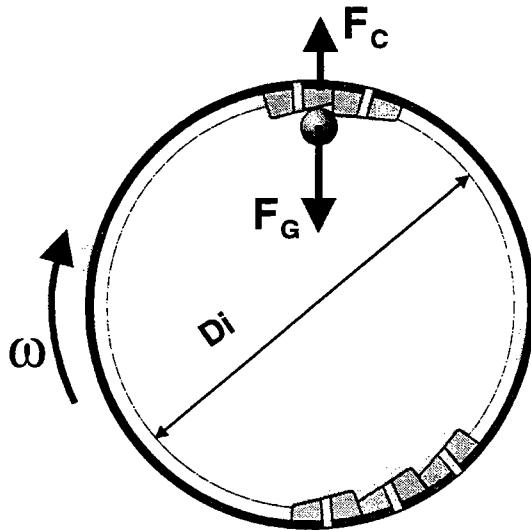
The operating mill speed  $n$  is then calculated as:

$$n = \frac{k}{100} \cdot n_{crit} \quad [\text{min}^{-1}] \quad (7)$$

<b>n</b>	=	operating mill speed	$[\text{min}^{-1}]$
<b>n<sub>crit</sub></b>	=	critical mill speed	$[\text{min}^{-1}]$
<b>k</b>	=	ratio $n/n_{crit}$	$[\%]$
<b>D<sub>i</sub></b>	=	internal mill diameter	$[\text{m}]$

Fig. 7: Critical Mill speed

**Fig.7 : Critical mill speed**



- FC =centrifugal force [N]
- FG =force of gravity [N]
- Di =mill diameter inside liner [m]
- ω =angular speed [s-1]
- g =gravity [ms-2]

$$n_{crit} = \frac{42,3}{\sqrt{D_i}} \quad [\text{min}^{-1}]$$

k = [%] of critical speed

$$n = k * n_{crit} \quad [\text{min}^{-1}]$$

The critical speed chart in Figure 8 is based on the equations 6 and 7 and allows a quick determination of the critical speed for tube mills with various diameters.

Figure 9 shows the ball charge behavior under different combinations of filling degrees and percentage of critical speed.

Low filling degrees and low percentage of critical speed do not allow an efficient ball charge action on the material. High filling degrees and high percentage of critical speed lead to ball charge centrifugation and very little grinding efficiency.

Mills with high percentage of critical speed can be operated with low filling degrees but wear of internal element is very high.

The framed picture in Fig. 9 shows a typical first chamber ball charge behavior for 30 [%] of filling degree and 70 [%] of critical speed.

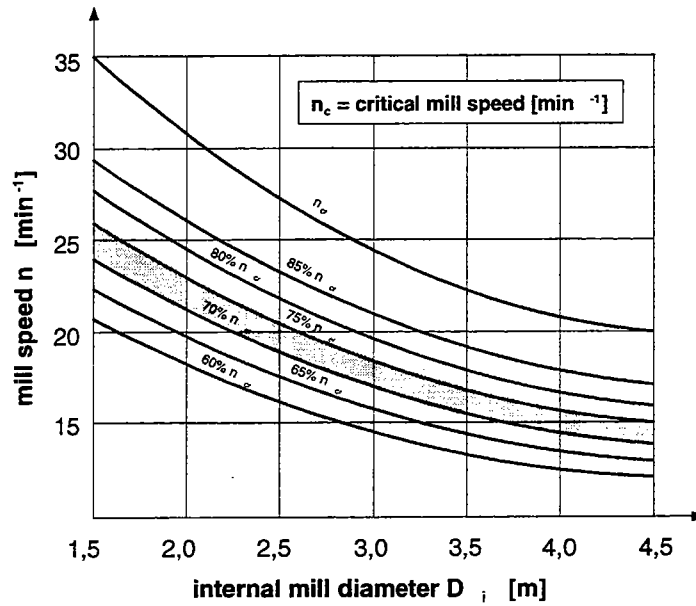
Grinding efficiency shows a somewhat indefinite peak in the range between 65 [%] and 75 [%] of critical speed. The driving power increases with mill speed. However, this linear relationship is only valid for the above mentioned range of speed. Therefore, an increase in out-put can be expected by increasing speed.

Modern mills have a speed range from 70 – 75 [%] of critical mill speed.

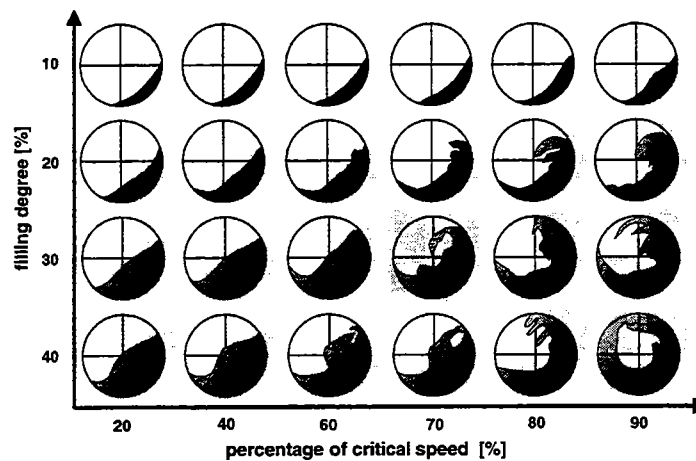
The type of liner, the filling degree and the ball charge composition are to be adjusted to the mill speed for optimum grinding performance.

Fig. 8 & 9

**Fig.8 : Percentage of critical mill speed in function of mill diameter**



**Fig.9 : Cascading effect of grinding media charge in function of filling degree and critical speed**



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### 3. NET DRIVING POWER

The net driving power for a tube mill is the power necessary at the mill shell to maintain the centre of gravity of the ball charge load in a position of kinetic equilibrium.

According to MITTAG the distance **b** (Figure 10) can be expressed as a function of the mill diameter  $D_i$ , taking into account the following assumptions:

- ◆ Distance **b** is always in the same relationship with the diameter for mills with the same filling degree but different diameters.
- ◆ Distance **b** does not depend on the mill speed for the usual range of speed.

The following formula can then be derived:

$$b = x \cdot D_i$$

The torque M can be expressed as:

$$M = x \cdot D_i \cdot Q \quad [Nm]$$

and the net driving power **P** as a function of the angular speed  $\omega$  and the torque **M**

$$P = M \cdot \omega \quad [kW]$$

With the angular speed  $\omega$  being

$$\omega = \frac{\pi \cdot n}{30} \quad [s^{-1}]$$

the net driving power P can finally be written as

$$P = \frac{x \cdot P_f \cdot Q \cdot \pi \cdot n}{30} \quad [kW]$$

For practical calculations the formula can be simplified to

$$P = c \cdot Q \cdot D_i \cdot n \quad [kW] \quad (8)$$

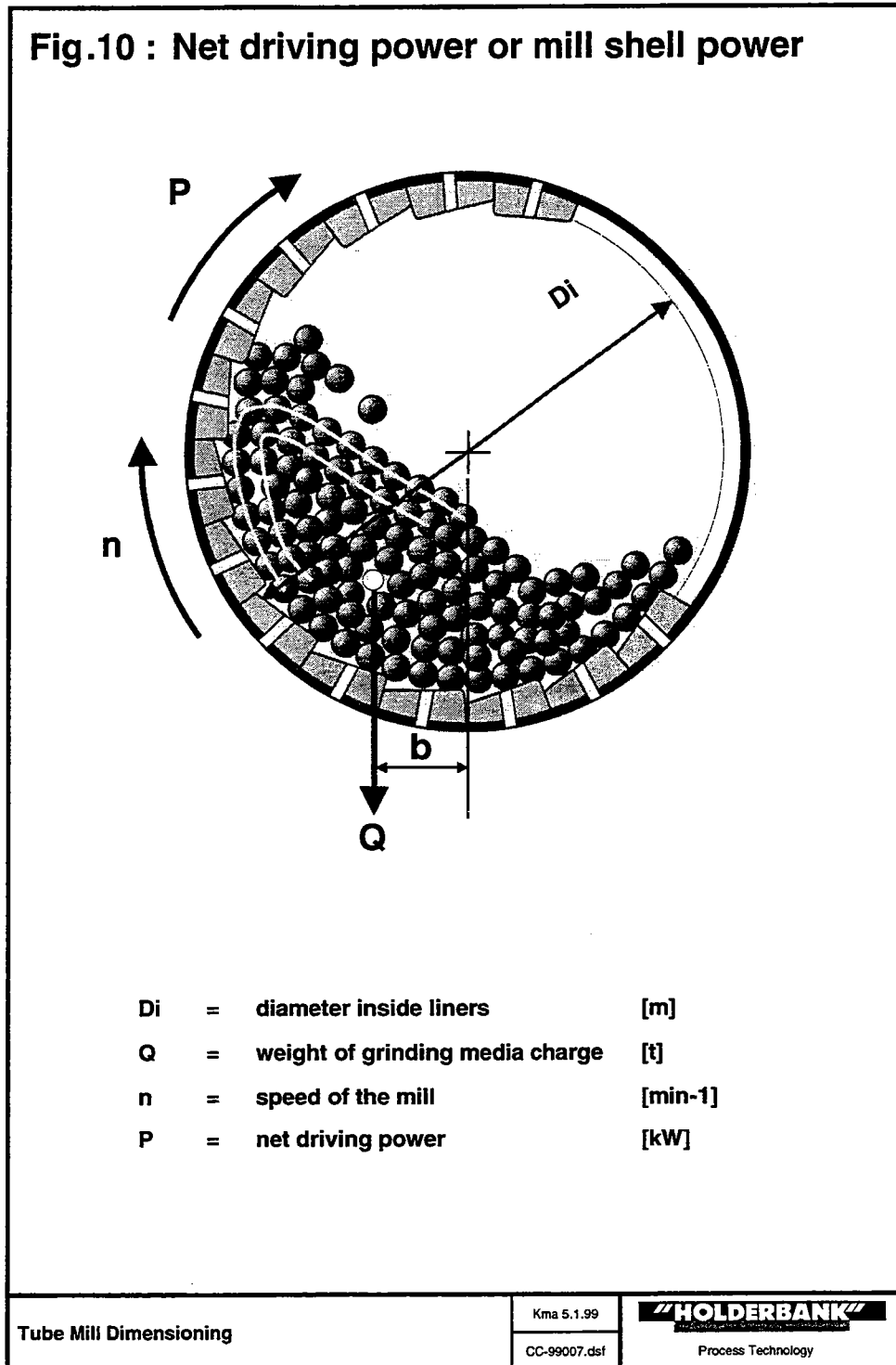
<b>P</b>	=	Net driving power	[kW]
<b>Q</b>	=	Grinding media charge	[t]
<b>D<sub>i</sub></b>	=	Mill diameter inside liners	[m]
<b>n</b>	=	Mill speed	[min <sup>-1</sup> ]
<b>c</b>	=	Power consumption factor	[-]

$$c = \frac{x \cdot \pi}{30}$$

Formula (8) allows the determination of the net driving power for a tube mill with an accuracy within 5 – 10 [%].

For multi-compartment mills the total net driving power can be calculated as the sum of driving powers for each individual compartment.

**Fig. 10:** Net driving power or mill shell power



The power consumption factor c depends on filling degree and on grinding media size. The value of  $x$  and therefore the value of  $c$  cannot be calculated theoretically. But by measuring the total driving power of industrial mills in operation and by considering the known values of  $Q$ ,  $D_i$  and  $n$ , the power consumption factor  $c$  can be determined.

Figure 11 shows this factor  $c$  versus the filling degree  $f$ .

The driving power is proportional to the:

- ◆ Weight of the grinding media charge
- ◆ Distance between the centre of gravity of the grinding media charge and the mill rotation centre, or lever.

As the filling degree increases, the weight of the grinding media increases but the lever decreases. It results that the driving power has a maximum value for a filling degree within 40 – 45 [%].

Formula (8) indicates that the net driving power varies as the grinding media charge, which in turn varies as the square of the diameter  $D_i$  for a given shell length; that driving power likewise varies as the diameter; and that it varies as the speed, which is an inverse function of the diameter, it follows that

$$P \approx D_i^2 \cdot D_i \frac{1}{\sqrt{D_i}} = D_i^{2.5}$$

This formula means that the net driving power of tube mills of the same length operating under the same relative conditions varies with 2.5 power of the internal diameter. Differences of length can be considered by the following formula

$$\frac{P_1}{P_2} = \frac{D_{i_1}^{2.5} \cdot L_{i_1}}{D_{i_2}^{2.5} \cdot L_{i_2}}$$

By assuming the same ratio of length to diameter for both mills the above formula can be written as:

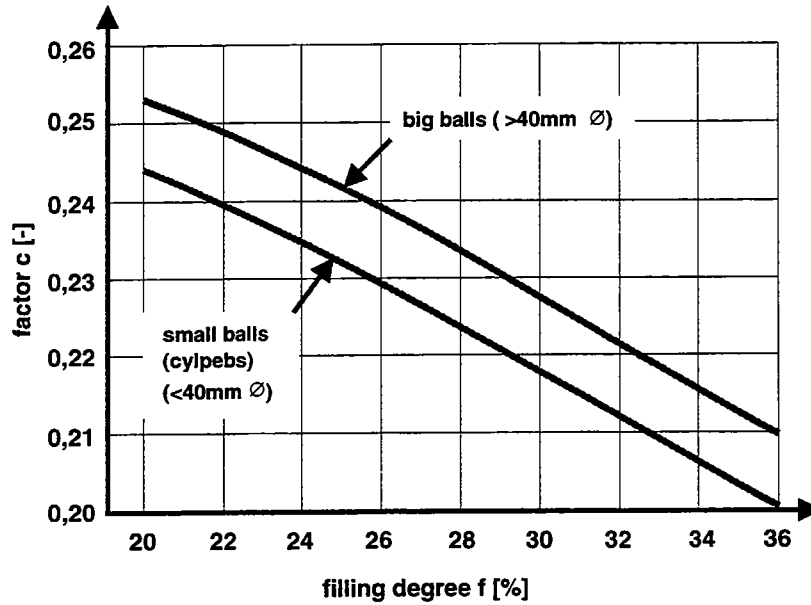
$$\frac{P_1}{P_2} = \left[ \frac{D_{i_1}}{D_{i_2}} \right]^{3.5} \quad (9)$$

It can be seen (from the above) that the diameter of a tube mill has a great influence ( $\sim D^{2.5}$ ) on the driving power, whereas the length of the mill has just linear influence (Figure 12).

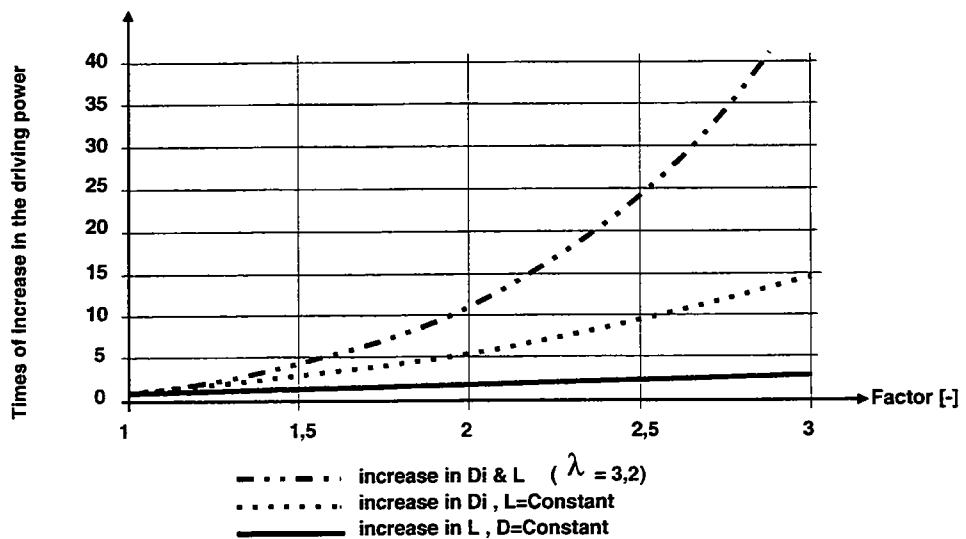


Fig. 11 & 12:

**Fig. 11 : Factor c, depending on filling degree and ball size**



**Fig. 12 : Increase of the net driving power of a tube mill with the Di and L**  
**Point (1,1) : Di=2, L=6.4**



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#### **4. POWER TRANSMISSION CHAIN**

Mill power is always dimensioned at mill motor shaft.

The power available for grinding is referred at the mill shell, also called net power. The power consumption the cement plant is billed for is measured at the mill motor kilowatt - hour meter. All these defined powers are lined to each other through the power transmission chain.

Figure 13 shows two possible transmission chains. The one in full line is for a mill with a central drive. The one in dotted line is for a mill with a gearth drive.

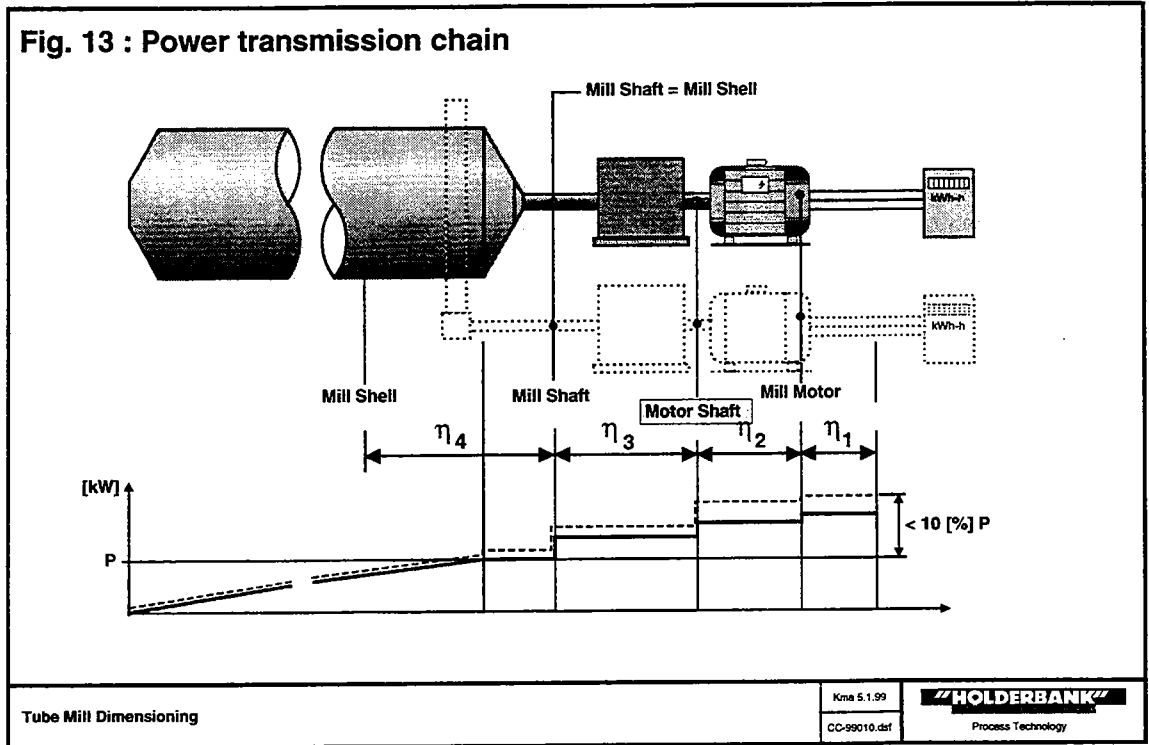
The efficiency figures shown are only indications and might vary with the type of mill drive and the mill power (see fig. 11 in the chapter “Tube Mills”).

$\eta_1$  to  $\eta_4$  are the efficiencies along the power transmission chain:

- ◆  $\eta_1$ : Due to cable loss between the mill motor and the power measuring point. Bigger than 99 [%] except if wattmeter very far.
- ◆  $\eta_2$ : Motor efficiency. (< 98 [%]).
- ◆  $\eta_3$ : Gearbox efficiency (< 97 [%]).
- ◆  $\eta_4$ : Pinion transmission efficiency. Bigger than 99 [%] if pinion in good state.

To avoid misunderstandings or in the worst case a wrong mill design it is of utmost importance to mention to which link of the chain the power is referred.

Fig. 13: Power transmission chain



**5. PRACTICAL CALCULATIONS**

For rough calculation of the main mill dimensions the mentioned formulas:

$$Q = \frac{\Pi}{4} \cdot D_i^2 \cdot L_u \cdot f \cdot \gamma_o \quad (2)$$

$$n_{crit} = \frac{42.3}{\sqrt{D_i}} \quad (6)$$

$$n = \frac{k}{100} \cdot n_{crit} \quad (7)$$

$$P = c \cdot Q \cdot D_i \cdot n \quad (8)$$

can be combined to one formula:

$$P = c \cdot Q \cdot D_i \cdot n = c \cdot \frac{\Pi}{4} \cdot D_i^2 \cdot L_u \cdot \frac{f}{100} \cdot \gamma_o \cdot \frac{42.3}{\sqrt{D_i}} \cdot \frac{k}{100}$$

$$P = 33.22 \cdot 10^{-4} \cdot c \cdot L_u \cdot f \cdot \gamma_o \cdot k \cdot D_i^{2.5}$$

Introducing the length to diameter ratio  $\lambda = \frac{L_u}{D_i}$  yields the following equation:

$$P = 33.22 \cdot 10^{-4} \cdot c \cdot \lambda \cdot f \cdot \gamma_o \cdot k \cdot D_i^{3.5} [kW] \quad (10)$$

The internal mill diameter can therefore be calculated according to formula (10) :

$$D_i = \left[ \frac{P}{33.22 \cdot 10^{-4} \cdot c \cdot \lambda \cdot f \cdot \gamma_o \cdot k} \right]^{1/3.5}$$

$$D_i = \left[ \frac{P}{33.22 \cdot 10^{-4} \cdot c \cdot \lambda \cdot f \cdot \gamma_o \cdot k} \right]^{0.286} \quad [m] \quad (11)$$

<b>P</b>	=	mill net driving power	[kW]
<b>c</b>	=	power consumption factor	[-]
<b>Q</b>	=	weight of grinding media charge	[t]
<b>L<sub>u</sub></b>	=	useful mill length	[m]
<b>D<sub>i</sub></b>	=	internal mill diameter	[m]
<b>k</b>	=	percentage of critical speed	[%]
<b>f</b>	=	filling degree	[%]
<b>γ<sub>o</sub></b>	=	bulk weight of grinding media	[t/m <sup>3</sup> ]
<b>λ</b>	=	length to diameter ratio	[-]